Studying caloric curves of nuclear matter

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Caloric curves measurements in heavy-ion collisions [1] have shown signs of a liquid-vapour phase transition in nuclear matter. The plateau region of the caloric curve, that is, the dependence of temperature, T, on excitation energy per particle ε_{ex}, gives the signal of phase transition [2]. For finite nuclear systems composed of a limited number of neutrons N and protons Z (small system), the extension of concepts as applied in the case of infinite matter is possible [3]. Generally, the liquid-vapour phase transition is accompanied by an increase of energy and particle number fluctuations. In this context it is of interest to consider fluctuations for thermodynamic states along the caloric curve. Here we address this issue within the grand canonical ensemble formulation.

The grand partition sum Ξ and grand potential Ω for the system driven by the corresponding environmental variables are written as,

$$\Xi(\lambda_0, \lambda_1, V, T) = \iint dA_0 dA_1 \exp\left[\left(\lambda_0 A_0 + \lambda_1 A_1 - F(A_0, A_1, V, T)\right)/T\right], \quad \Omega = -T\ln(\Xi). \tag{1}$$

Here, the integration is carried out over possible values of total number of nucleons $A_0 = N + Z$ and neutron excess $A_1 = N - Z$. In Eq. (1), $F = F(A_0, A_1, V, T)$ stands for the free energy, λ_0 and λ_1 are, respectively, the isoscalar and isovector chemical potentials, V is the system volume, and T is the temperature. In order to calculate the average quantities and fluctuations, the probability distribution function $p(A_0, A_1) = \exp[(\lambda_0 A_0 + \lambda_1 A_1 - F)/T]/\Xi$ is introduced. Using this distribution function the average values of particle number, $\langle A_0\rangle$, neutron excess, $\langle A_1\rangle$, pressure, $\langle P\rangle$, and energy, $\langle E\rangle$, are reduced to the first derivatives of the grand potential (1) for λ_0, λ_1, V and T, respectively,

$$\langle A_0\rangle = -\left(\frac{\partial\Omega}{\partial\lambda_0}\right)_{\lambda_1,V,T}, \quad \langle A_1\rangle = -\left(\frac{\partial\Omega}{\partial\lambda_1}\right)_{\lambda_0,V,T}, \quad \langle P\rangle = -\left(\frac{\partial\Omega}{\partial V}\right)_{\lambda_0,\lambda_1,T},$$

$$\langle E\rangle = \Omega - T\left(\frac{\partial\Omega}{\partial T}\right)_{\lambda_0,\lambda_1,V} - \lambda_0\left(\frac{\partial\Omega}{\partial A_0}\right)_{\lambda_1,V,T} - \lambda_1\left(\frac{\partial\Omega}{\partial A_1}\right)_{\lambda_0,V,T}. \tag{2}$$

The excitation energy per particle ε_{ex}, needed for determination of the caloric curve, $T(\varepsilon_{\text{ex}})$, is obtained from Eq. (2) as

$$\varepsilon_{\text{ex}} = (\langle E\rangle - E_{gs})/\langle A_0\rangle, \tag{3}$$

where E_{gs} is the ground state energy at $T = 0$. One should note that, for the considered small system, the energy $\langle E\rangle$ is not linear homogeneous function of entropy $\langle S\rangle$, volume V, and $\langle A_0\rangle$, $\langle A_1\rangle$ numbers, in contrast to the macroscopic limit $V \to \infty$ (within the habitual thermodynamics for infinite nuclear matter) when all extensive properties become functions of λ_0, λ_1, and T only. Nevertheless, for the certain set of environment variables, like λ_0, λ_1, V and T in our case, the thermodynamics of small system can be built [4]. Along with "differential" pressure $\langle P \rangle$, see Eq. (2), the "integral" pressure $\hat{P} = -\Omega/V$ is introduced, and the average energy takes the form $\langle E \rangle = T\langle S \rangle - \langle P \rangle V + \lambda_0 \langle A_0 \rangle + \lambda_1 \langle A_1 \rangle + \mathcal{E}$, with $\mathcal{E} = \left(\langle P \rangle - \hat{P} \right) V$ being the correction term for the small system which disappears in the macroscopic limit, i.e. $-(\partial \Omega / \partial V)_{\lambda_0, \lambda_1, T} = -\Omega/V = \hat{P} = \langle P \rangle$ (see Ref. [4]).

We have calculated the isobaric caloric curve, using $\hat{P} = 0.05\ \mathrm{MeV/fm^3}$, for the small nuclear system having on the average $\langle A_0 \rangle = 200$ and $\langle A_1 \rangle = 40$ and the asymmetry parameter $X = \langle A_1 \rangle / \langle A_0 \rangle = 0.2$. Calculation was carried out for the temperature interval $T = 5 \div 12$ MeV using KDE0v1 Skyrme effective nucleon-nucleon interaction [5]. At each chosen temperature the values of λ_0, λ_1, and V were determined to provide the above mentioned values of $\langle A_0 \rangle$, $\langle A_1 \rangle$, and $\hat{P}$. The result is shown in Fig. 1 by the dotted line. With the aim of comparison, the calculation at the same pressure and asymmetry parameter was carried out for infinite asymmetric nuclear matter (solid and dashed lines in Fig. 1). Comparing the dotted line with the solid one in Fig. 1 it is seen that the temperature in the middle of plateau region for the small system is lower than that for infinite matter by about of 0.2 MeV.

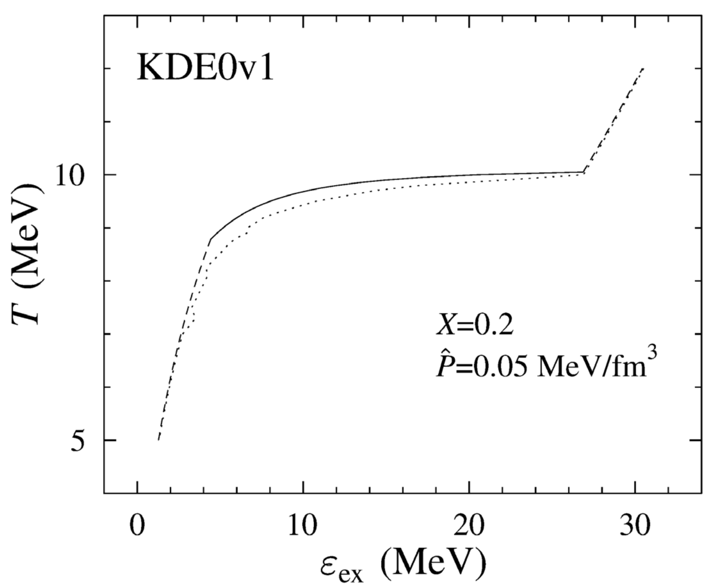

FIG. 1. Isobaric caloric curves $\boldsymbol{T(\varepsilon_{\mathrm{ex}})}$. Dotted line presents the result at pressure $\boldsymbol{\hat{P} = 0.05\ \mathrm{MeV/fm^3}}$ for small nuclear system with $\boldsymbol{\langle A_0 \rangle = 200}$, $\boldsymbol{\langle A_1 \rangle = 40}$. Solid and dashed lines shows the result in the case of infinite matter for the same pressure and asymmetry parameter. Dashed line correspond to a single phase, solid line is obtained for phase coexistence region. Calculations were carried out using KDE0v1 Skyrme nucleon-nucleon effective interaction [5].

We also obtained the relative fluctuations of the nucleon number, δ_0, and the neutron excess, δ_1, by means of the following expressions

$$\delta_0 = \frac{\left(\langle A_0^2\rangle - \langle A_0\rangle^2\right)^{1/2}}{\langle A_0\rangle} = -\frac{\sqrt{-T\left(\frac{\partial^2 \Omega}{\partial \lambda_0^2}\right)_{\lambda_1,V,T}}}{\left(\frac{\partial \Omega}{\partial \lambda_0}\right)_{\lambda_1,V,T}} \;,\quad \delta_1 = \frac{\left(\langle A_1^2\rangle - \langle A_1\rangle^2\right)^{1/2}}{\langle A_1\rangle} = -\frac{\sqrt{-T\left(\frac{\partial^2 \Omega}{\partial \lambda_1^2}\right)_{\lambda_0,V,T}}}{\left(\frac{\partial \Omega}{\partial \lambda_1}\right)_{\lambda_0,V,T}} \;. \tag{4}$$

The calculation of the dispersion and, consequently, the fluctuation (absolute or relative) of A_0 and A_1 requires the value of the second derivative of the grand potential Ω with respect to the corresponding chemical potential. Fig. 2 presents the relative fluctuations δ_0 and δ_1 for small nuclear system $\langle A_0\rangle = 200$, $\langle A_1\rangle = 40$ as functions of excitation energy per particle. Fig. 2 demonstrates the increase of fluctuations in the two-phase region of excitation energies. Such an increase, together with the plateau region in caloric curve $T(\varepsilon_{\text{ex}})$, gives the signature of the occurring phase transition.

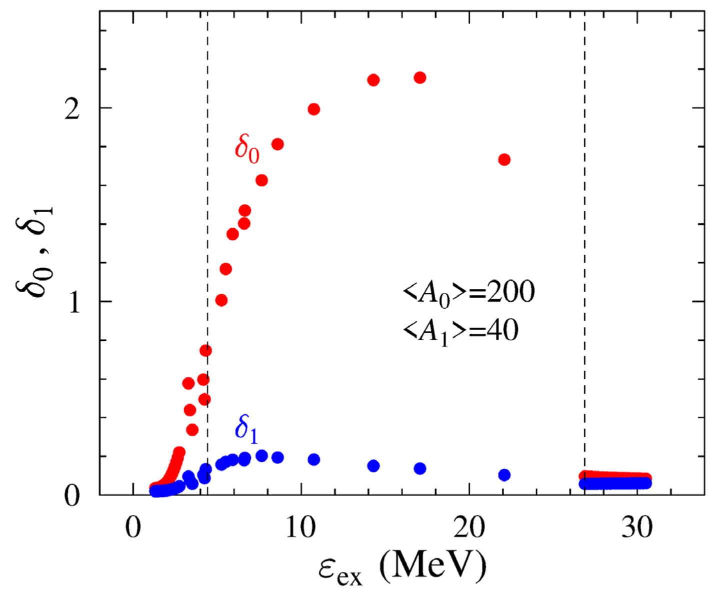

FIG. 2. Relative fluctuations of the nucleon number $\boldsymbol{\delta_0}$ (red dots) and neutron excess $\boldsymbol{\delta_1}$ (blue dots) versus the excitation energy per nucleon $\boldsymbol{\varepsilon_{\text{ex}}}$, see Eqs. (3), and (4). Results are obtained for small nuclear system with $\langle \boldsymbol{A_0}\rangle = \mathbf{200}$ and $\langle \boldsymbol{A_1}\rangle = \mathbf{40}$ along the caloric curve, see Fig. 1. The range of $\boldsymbol{\varepsilon_{\text{ex}}}$ between vertical dashed lines corresponds to coexistence of liquid and vapour phases for the case of infinite nuclear matter.

In spite of the presented results for small nuclear system do not include the effects of Coulomb interaction and nuclear surface, they still can be valuable to give an idea on the excitation energy range where to expect the observation of liquid-vapour phase transition.

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